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Analysis of occurrence of digit 0 in first 10 billion digits of π after decimal point

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Abstract: **π has fascinated mathematicians from ancient era. In fact, every irrational number has that potential owing to the non-recursive pattern of infinite non-zero digits after the decimal point and their random occurrence. The present work is another attempt to unveil this mystery by analyzing the occurrence of digit 0 in first 10 billion digits of π after Decimal Point in Decimal Representation. Both successive and non-successive occurrences of 0 in π have been extensively analyzed.**

Keywords: **Digit 0, Digits after decimal point, π** ***Mathematics Subject Classification 2010:*** **11Y35, 11Y60, 11Y99.**

1. INTRODUCTION

Amongst the real numbers, the irrational numbers enjoy mysterious status. They have infinite non-repeating non-zero digits after the decimal point, and hence we lack knowledge of their precise values. We have always to settle down to some approximation accepting only a few correct digits and neglecting infinitely many after those. The apparent randomness in the digit sequence of the irrational numbers makes them perfect for many applications purposefully demanding this nature.

We choose the well-known candidate π [1]. As all are aware, this frequently occurs in geometry in connection with circle and related shapes, and also in trigonometry, analysis, and many modern branches. It has been studied extensively by great mathematicians like Archimedes, Newton, Euler, John von Neumann, and Ramanujan.

2. Digit 0 in π

There has been lot of work on the digits of π [2]. We have exhaustively analyzed the occurrence of 0 in the digits of π after its decimal point. Such kind of analysis of occurrence of digit 1 in natural numbers has been recently done [3]. Here, we have taken into consideration as many as 10 billion digits of π after the decimal point. Since we usually use the decimal system with base 10, considering the ranges of $1–10^x$, for $1 \le x \le 10$, following results have been derived.

TABLE.I: OCCURRENCES OF DIGIT 0 IN BLOCKS OF 10 POWERS

Sr. No.	Digit Numbers' Range $1 – x$	Range as Ten Power 10^x	Number of Occurrences of 0	First Occurrence of 0 at Digit Number	Last Occurrence of 0 at Digit Number
1.	1 – 10	10^1	0	-	-
2.	1 – 100	10^2	8	32	97
3.	1 – 1,000	10^3	93	32	996
4.	1 – 10,000	10^4	968	32	9,987
5.	1 – 100,000	10^5	9,999	32	99,987
6.	1 – 1,000,000	10^6	99,959	32	999,990
7.	1 – 10,000,000	10^7	999,440	32	9,999,979
8.	1 – 100,000,000	10^8	9,999,922	32	99,999,991
9.	1 – 1,000,000,000	10^9	99,993,942	32	999,999,995
10.	1 – 10,000,000,000	10^{10}	999,967,995	32	10,000,000,000

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In the very first 10 power block 1 – 10, the digit 0 doesn't occur even once. It appears first late at digit number 32.

Had all 10 digits from 0 to 9 evenly occurred in all ranges, the expected average of occurrence of each digit would have been one tenth of the range-limit. With this unbiased expectation as base, the deviation from the average for occurrence of 0 is as shown below.

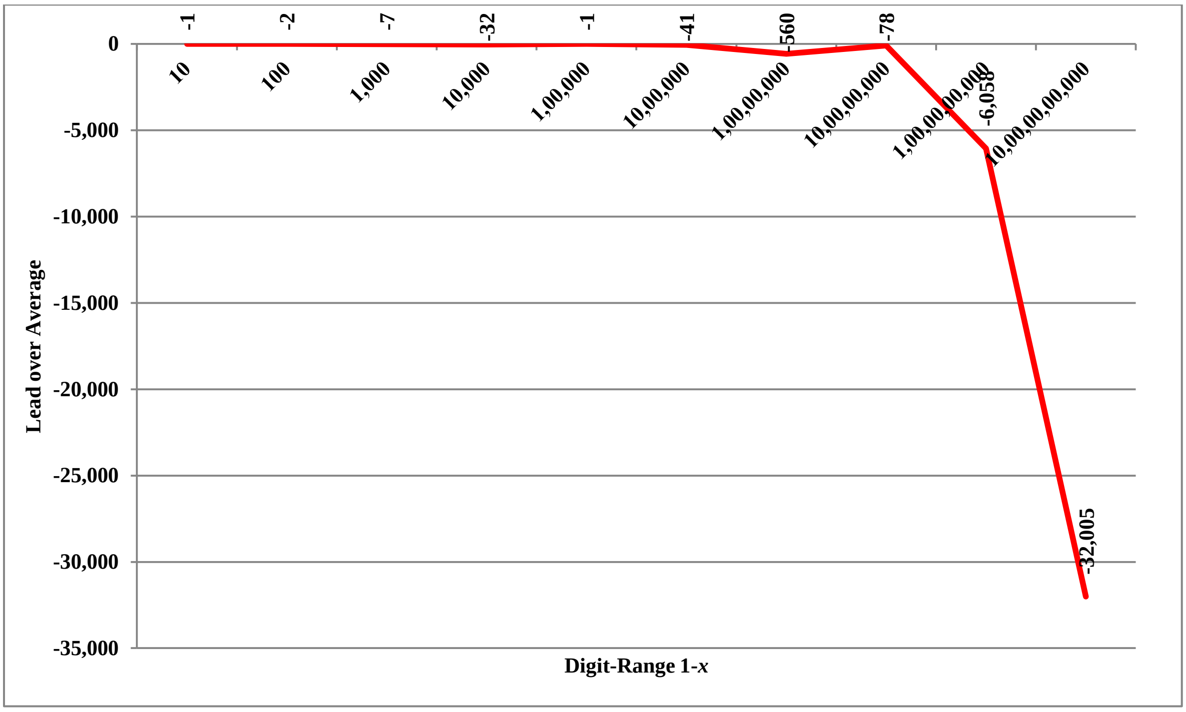

FIGURE.I: DEVIATION FROM THE AVERAGE FOR OCCURRENCE OF 0 IN BLOCKS OF 10^n

As far as the above 10 discrete digit range-vales are considered, 0 seems always below average. Whether this is consistent behavior for all higher ranges is a subject matter of further investigation. Also for values in between these ranges, 0 does take lead over average many times, but we have considered only discrete ranges.

As noted earlier, first 10 digits of π do not contain 0. But it occurs at digit number 32 and is the first occurrence value for all higher ranges.

Barring the first block of 1 – 10, the last occurrence of digit 0 falls short to reach the last digit by following amounts.

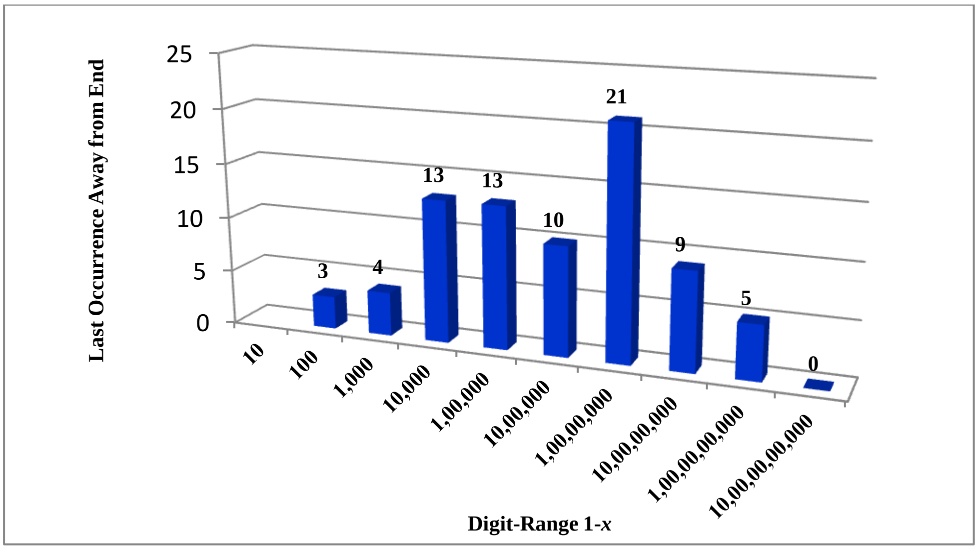

FIGURE II: DISTANCE OF LAST OCCURRENCE OF 0 IN BLOCKS OF 10^n FROM END

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Quantitatively, 0 is farthest from end in block of $1 – 10^7$ and nearest in block of $1 – 10^{10}$, in fact it is right at the end of the block there.

3. Successive Occurrence of Digit 0 in π

The successive occurrence of digit 0 has also been under rigorous investigation.

TABLE.II: SUCCESSIVE OCCURRENCES OF DIGIT 0 IN BLOCKS OF 10 POWERS

Sr. No.	Digit Numbers' Range 1 – *x*	Range as Ten Power 10^x	Number of Successive Occurrences of 0	First Successive Occurrence of 0 at Digit Number	Last Successive Occurrence of 0 at Digit Number
1.	1 – 10	10^1	0	-	-
2.	1 – 100	10^2	0	-	-
3.	1 – 1,000	10^3	7	307	973
4.	1 – 10,000	10^4	85	307	9984
5.	1 – 100,000	10^5	998	307	99,854
6.	1 – 1,000,000	10^6	9,938	307	999,769
7.	1 – 10,000,000	10^7	99,662	307	9,999,687
8.	1 – 100,000,000	10^8	999,781	307	99,999,894
9.	1 – 1,000,000,000	10^9	10,004,524	307	999,999,679
10.	1 – 10,000,000,000	10^{10}	100,000,652	307	9,999,999,975

In the first two 10 power blocks 1 – 10 and 1 – 100, digit 0 doesn't occur consecutively even once. It starts occurring such from digit number 307.

The percentage of occurrence of successive 0's in respective blocks has following slight initial rising trend.

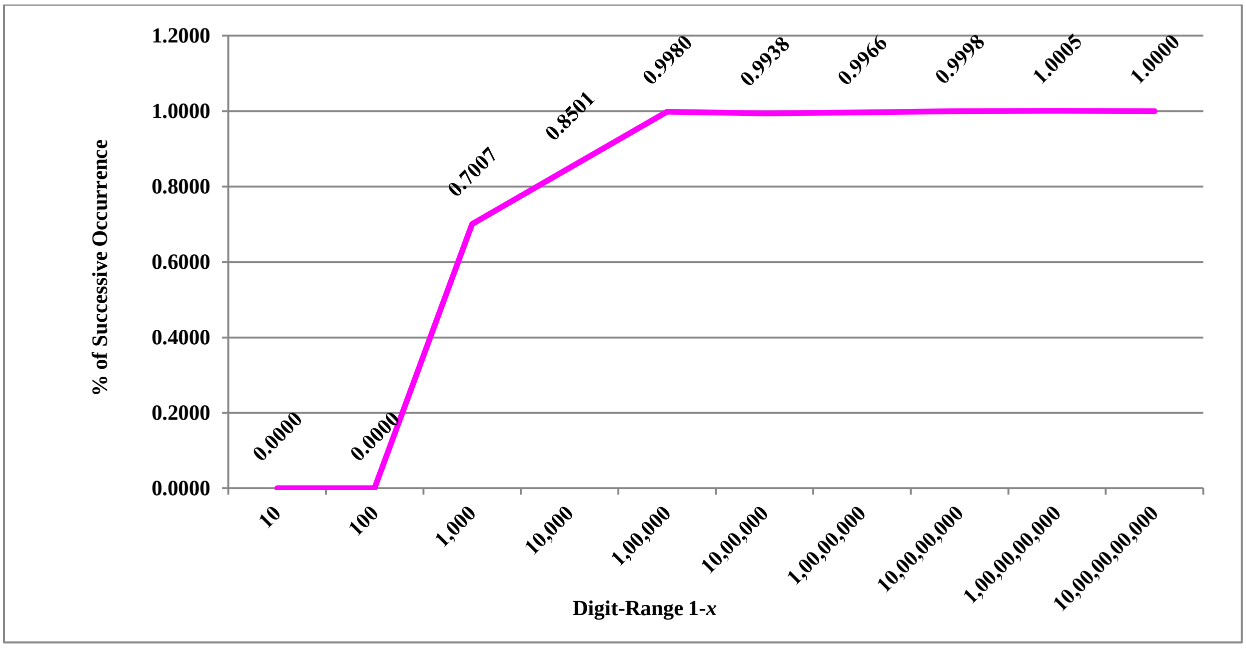

FIGURE.III: PERCENTAGE OF SUCCESSIVE OCCURRENCES OF 0'S IN BLOCKS OF 10^n

The percentage smoothly reaches unity. The first successive occurrence of 0 comes in late; more than 9 times of solo occurrence.

Barring the first two blocks of 1 – 10 & 1 – 100, the beginning of last successive occurrence of digit 0's stops before last digit in range by keeping following spaces.

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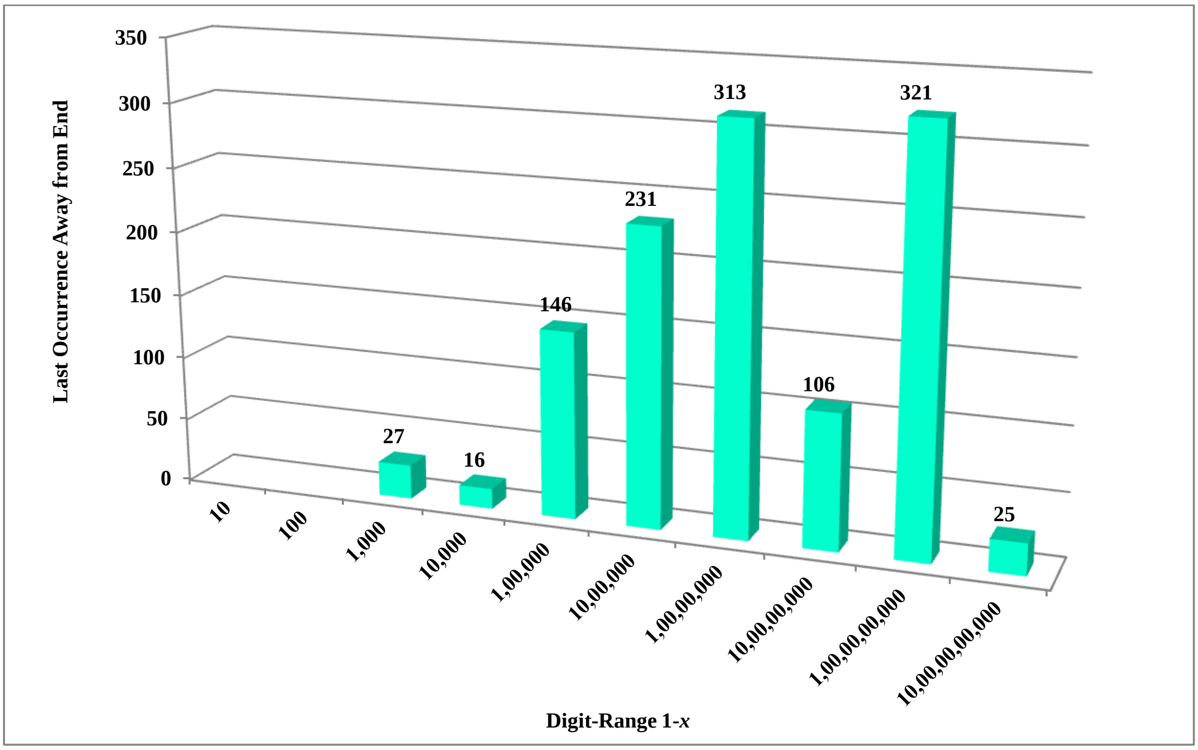

FIGURE.IV: DISTANCE OF LAST SUCCESSIVE OCCURRENCE OF 0 IN BLOCKS OF 10^n FROM END

No specific regularity is apparent.

The above analysis has been for 2 consecutive 0's. As many as 10 consecutive occurrences of 0's are found in these digit ranges. Their exact tally is as follows.

TABLE.III: MULTIPLE SUCCESSIVE OCCURRENCES OF DIGIT 0 IN BLOCKS OF 10 POWERS

Sr. No.	Digit Range	Number of Successive 0's									
		1	2	3	4	5	6	7	8	9	10
1.	$1\text{-}10^{1}$	0	0	0	0	0	0	0	0	0	0
2.	$1\text{-}10^{2}$	8	0	0	0	0	0	0	0	0	0
3.	$1\text{-}10^{3}$	93	7	2	0	0	0	0	0	0	0
4.	$1\text{-}10^{4}$	968	85	7	0	0	0	0	0	0	0
5.	$1\text{-}10^{5}$	9,999	998	102	9	1	0	0	0	0	0
6.	$1\text{-}10^{6}$	99,959	9,938	967	96	6	0	0	0	0	0
7.	$1\text{-}10^{7}$	999,440	99,662	9,877	902	89	6	1	0	0	0
8.	$1\text{-}10^{8}$	9,999,922	999,781	99,746	9,818	991	90	8	0	0	0
9.	$1\text{-}10^{9}$	99,993,942	10,004,524	1,000,897	99,631	9,968	941	92	8	0	0
10.	$1\text{-}10^{10}$	999,967,995	100,000,651	10,001,867	1,000,885	100,228	10,126	1,005	112	11	1

In any specific digit block of $1 - 10^x$, during existence, the rate of recurrence of increasing sequence of 0's fades out roughly by order of magnitude 10.

The first instances of these successive patterns of 0's are approximated by $y = 1.5542e^{2.2439x}$, which is an exponential function.

As the specific number of successive instances of digit 0 goes on increasing at a higher rate, we have taken vertical axis on the logarithmic scale. Due to this reason, the trend line(!) looking as line in the following semi-logarithmic graph is actually an exponential-like curve!

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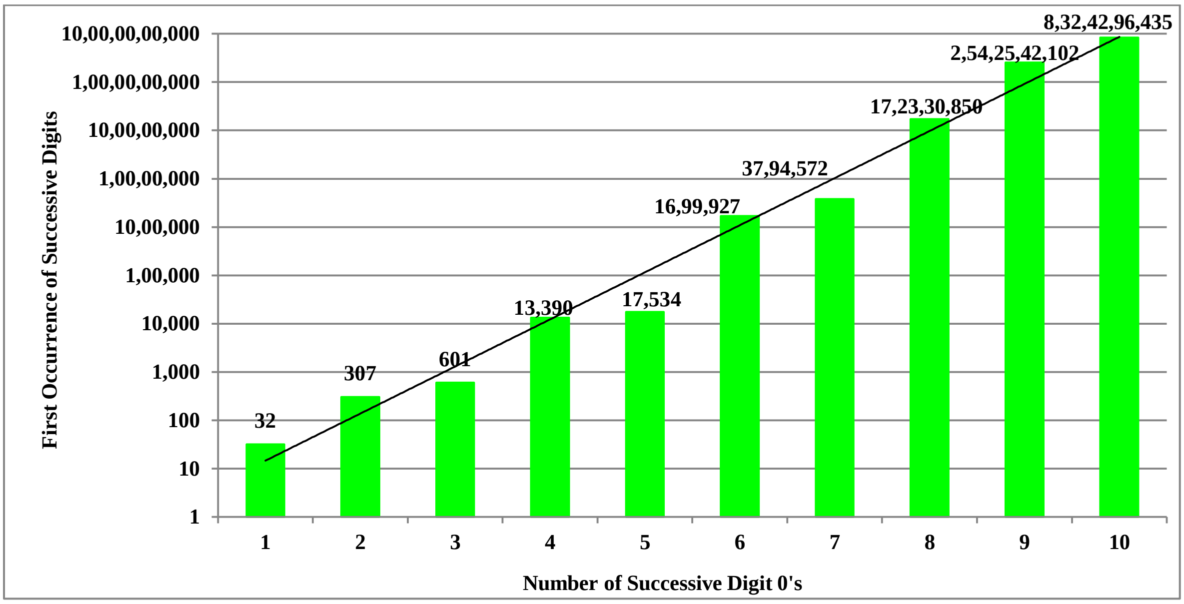

FIGURE.V: FIRST MULTIPLE SUCCESSIVE OCCURRENCES OF 0 IN BLOCKS OF 10^n

The last occurrences of multiple successive 0's in the blocks of 10^x are determined to be following.

TABLE.IV: LAST MULTIPLE SUCCESSIVE OCCURRENCES OF DIGIT 0 IN BLOCKS OF 10 POWERS

0's	Digit Range and Last Occurrence									
↓	10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8	10^9	10^{10}
1	-	97	996	9,987	99,987	999,990	9,999,979	99,999,991	999,999,995	10,000,000,000
2	-	-	973	9,984	99,854	999,769	9,999,687	99,999,894	999,999,679	9,999,999,975
3	-	-	855	8,879	99,754	999,092	9,998,146	99,999,858	999,998,672	9,999,998,260
4	-	-	-	-	93,041	984,027	9,993,613	99,996,294	999,998,671	9,999,997,359
5	-	-	-	-	17,534	967,625	9,968,412	99,982,598	999,888,798	9,999,975,941
6	-	-	-	-	-	-	7,257,528	98,289,246	995,372,837	9,999,132,417
7	-	-	-	-	-	-	3,794,572	98,096,446	951,988,581	9,992,939,703
8	-	-	-	-	-	-	-	-	627,213,906	9,907,959,702
9	-	-	-	-	-	-	-	-	-	8,324,296,436
10	-	-	-	-	-	-	-	-	-	8,324,296,435

4. Non-Consecutive Occurrence of Digit 0 in π

The occurrence of digit 0 with other digit(s) in between it and next 0 has also been determined. In this process such an occurrence is counted only if the next 0 falls in the same block otherwise it is not credited for that block.

TABLE.V: NON-SUCCESSIVE OCCURRENCES OF DIGIT 0 IN BLOCKS OF 10 POWERS

Sr. No.	Digit Numbers' Range 1 – *x*	Range as Ten Power 10^x	Number of Non-Successive Occurrences of 0	First Non-Successive Occurrence of 0 at Digit Number	Last Non-Successive Occurrence of 0 at Digit Number
1.	1 – 10	10^1	0	-	-
2.	1 – 100	10^2	7	32	85
3.	1 – 1,000	10^3	85	32	989
4.	1 – 10,000	10^4	882	32	9,985
5.	1 – 100,000	10^5	9,000	32	99,985
6.	1 – 1,000,000	10^6	90,020	32	999,987
7.	1 – 10,000,000	10^7	899,777	32	9,999,964
8.	1 – 100,000,000	10^8	9,000,140	32	99,999,985
9.	1 – 1,000,000,000	10^9	89,989,417	32	999,999,972
10.	1 – 10,000,000,000	10^{10}	899,967,342	32	9,999,999,983

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In the first 10 power block, digit 0 doesn't occur non-consecutively. It starts occurring such from digit number 32.

The instances of non-consecutive occurrences of 0 dominate those of consecutive occurrences.

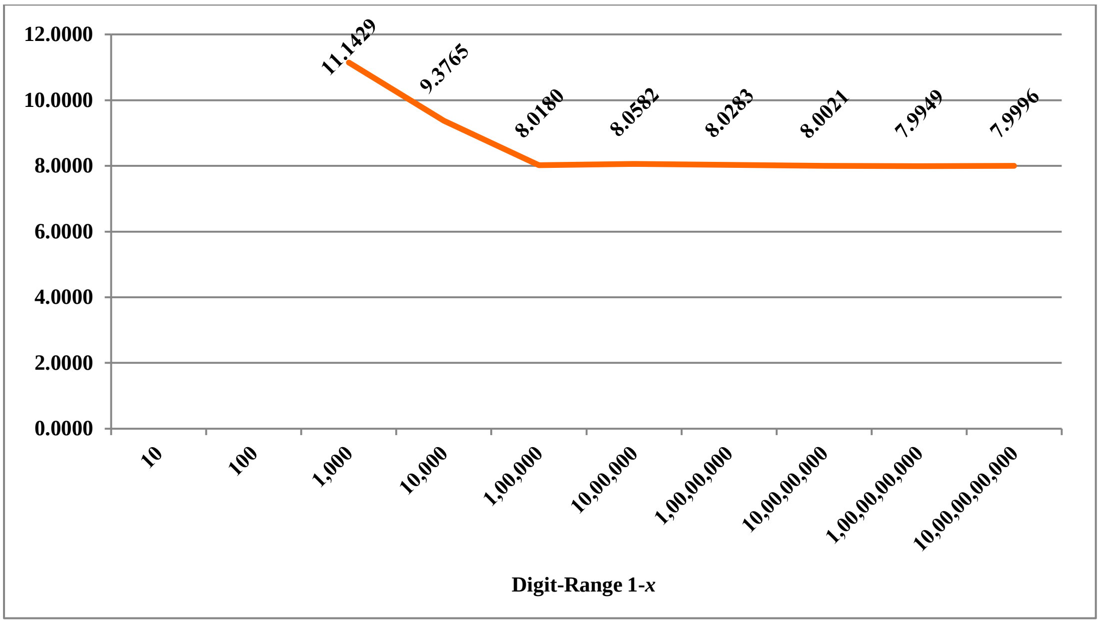

FIGURE.VI: NUMBER OF TIMES NON-SUCCESSIVE OCCURRENCES ARE MORE THAN CORRESPONDING SUCCESSIVE OCCURRENCES OF 0

The first non-consecutive occurrence of digit 0 matches with that of the very first occurrence.

Exempting first block of 1 – 10 of non-occurrence, the last non-successive occurrence of digit 0 halts prior to last digit in range by following quantities

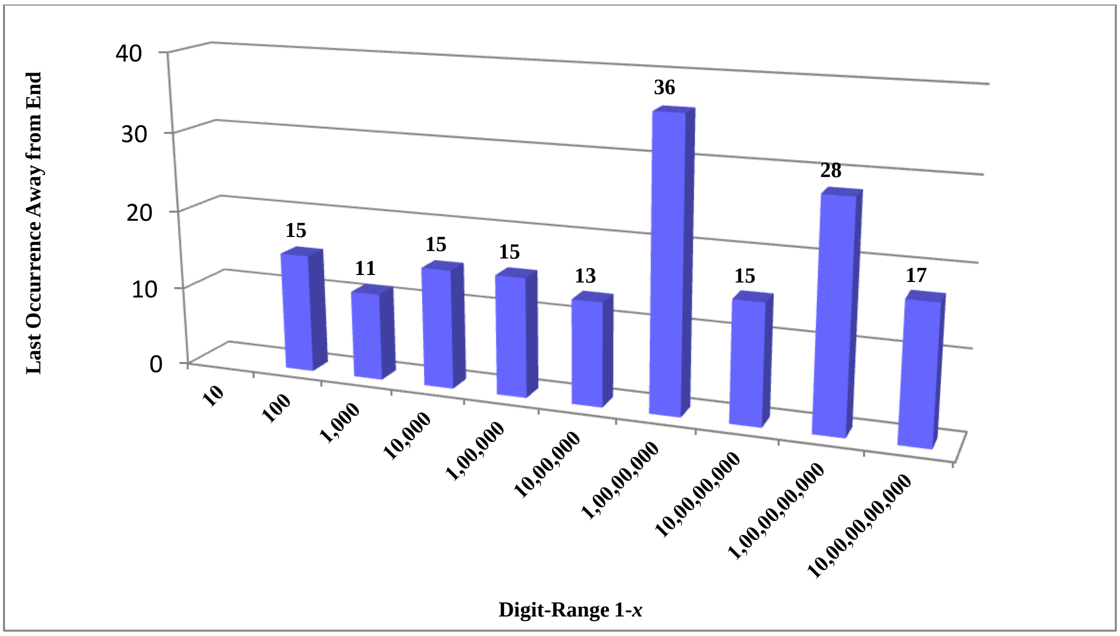

FIGURE.VII: DISTANCE OF LAST NON-SUCCESSIVE OCCURRENCE OF 0 IN BLOCKS OF 10^n FROM END

Higher frequency of non-consecutive occurrences of digit 0 naturally pulls down the heights of these bars as compared to their counterparts for successive ones.

This analysis has been an attempt to recognize patterns and/or regularity, if any, in decimal digits of π through occurrence of digit 0 in them. Future plans involve treatment of few other digits also.

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